Journal of Communications vol. 18, no. 1, January 2023

A Modified Selective Mapping Technique for Peak-to-Average Power Ratio Reduction in Orthogonal Frequency Division Multiplexing with Index Modulation Systems

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Abstract**—**In this study, we propose a modified selective mapping (SLM) technique to reduce the Peak-to-Average Power Ratio (PAPR) of Orthogonal Frequency Division Multiplexing with Index Modulation (OFDM-IM) systems. The main reason for using Reed–Muller and Polar codes is to protect the information bits to be transmitted while providing error correction capability and improved PAPR performance to the transmitted signal. Although OFDM-IM technology is a popular communication transmission technology recently, one of its disadvantages is that the transmitted signal has a high PAPR. The proposed method uses the Hamming weight distribution of the specific order Reed–Muller code to create a corresponding lookup table to select the subcarriers that are active in the OFDM-IM system. Moreover, it can correct errors in the information used. The proposed method also uses Polar codes as a channel encoder of the OFDM-IM system to analyze its performance in improving the PAPR. The simulation results show that method can not only improve the PAPR performance of OFDM-IM systems but also provide OFDM-IM transmission-signal error correction capacity.

Index Terms**—**Orthogonal Frequency Division Multiplexing with Index Modulation (OFDM-IM), peak-to-average power ratio, reed–Muller codes, polar codes

I. INTRODUCTION

In recent years, the amount of communication transmission has increased dramatically because of the rapid development of intelligent communication engineering, and the Orthogonal Frequency Division Multiplexing (OFDM) technology with a high transmission data rate has also received attention. OFDM technology is an important transmission technology in the fourth generation of cell phone mobile communication technology (4G) standards and has been widely used in various wireless communication standards, including Wi-Fi, IEEE 802.11 a/g, 802.16 WiMAX, and Long-Term Evolution (LTE) [1]. However, with the fast growth of mobile data services and the popularity of smart devices, people are interested in achieving frequency spectrum and energy efficiency, taking 5G networks for example. Given the evolution of this trend, OFDM with index modulation (OFDM-IM) technology has become a hot research topic recently. The main reason is that the index modulation can maintain high data throughput and low energy expenditure [2]-[5]. Unlike OFDM technology, not all OFDM-IM subcarriers are used, but some subcarriers can be designated by an index to transmit data, and subcarriers that are not designated remain unused. Additionally, the side information used to describe the content of the indicator is implicit in the subcarriers used to transmit the data.

As stated above, index modulation can improve energy efficiency. The main reason is that, in addition to the information bits carried by digital modulation transmission, it can transmit the side information implied by the index without consuming energy. Although OFDM has many advantages, a high Peak-to-Average Power Ratio (PAPR) still occurs in OFDM signals. Additionally, high-power amplifiers are prone to distortion and can increase the complexity of analog to digital converters (A/D converters) [1]. An OFDM with an index modulation technique also has this disadvantage. Many methods have been proposed in the literature [6]–[13].

The author of [9] discussed on the performance of a system to improve the PAPR of the selective mapping (SLM) technology in the phase generation mechanism, as well as the arrangement and combination of the input data. In [9], the authors analyze and discuss two parts: the first part is to design the permutation and combination mechanism based on the number of active subcarriers set by the OFDM-IM system, and the second part is to use the permutation and combination mechanism to generate the set of phase sequences. The simulation results in [9] show that using the permutation combination only when the number of active subcarriers of each subblock is small improves the PAPR performance better than not using the permutation combination. However, when the number of active subcarriers of each subblock is increased, the improved PAPR performance of both methods is comparable. The authors also found that using the row vector of the cyclic Hadamard matrix as the phase sequences of a traditional SLM technique can improve PAPR performance when the number of active subcarriers of each subblock is increased. Although the authors propose subcarriers permutation, the combination mechanism and the generation conditions of the optimal phase sequence set, how to further improve the PAPR performance remains an unsolved problem. Therefore,

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Manuscript received June 13, 2022; revised December 25, 2022; accepted January 2, 2023.

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doi:10.12720/jcm.18.1.76-81

this study proposes a modified SLM technique to improve the PAPR performance of the OFDM-IM system and enable the transmitted signal to have an error correction capacity.

The method proposed in this study uses two linear block codes to protect the input data of the OFDM-IM system, where one linear block code is used to protect the data of the active subcarriers and the other is used to protect the transmitted data. Particularly, the data of the active subcarrier can be protected by selecting the position of the active subcarriers based on the codeword of the Reed–Muller (RM) codes. The Polar code is also used as the channel coding to protect the transmitted data. The simulation results show that the proposed method not only improves the PAPR performance over the reference [9] and SLM techniques used in OFDM-IM systems but also enables the transmitted signal to have an error correction capacity simultaneously.

The remainder of the study is structured as follows. Section II details the mathematical expressions of OFDM-IM signals, the definition of PAPR, and the SLM techniques. Section III describes the system architecture of the proposed method. Sections IV and V present the simulation results and the conclusion, respectively.

II. OFDM-IM SYSTEM

A. OFDM-IM

Fig. 1 shows a block diagram of the transmitter of an OFDM-IM system. An input digital data sequence with s bits is first divided into g groups with p bits using a bit splitter (Fig. 1), where $s = gp$. Furthermore, the OFDM-IM system has N subcarriers, and the N subcarriers are divided into g subgroups with L subcarriers, where $N = gL$. Assuming that each subgroup has $p = p_1 + p_2$ data bits, these bits can be divided into two parts as follows:

$$p_1 = \left\lfloor \log_2 C_k^L \right\rfloor,\ C_k^L = \frac{L!}{(L-k)!\ k!} \tag{1}$$

bits and

$$p_2 = k \log_2 M \tag{2}$$

bits, where $k! = k \times (k-1) \times (k-2) \times \cdots \times 1$. The p_1 bits used as the input bits of the index selector are used to determine which k subcarriers are the active subcarriers of the subgroup with L subcarriers; the p_2 bits are input to the constellation mapper, which is used to modulate the bits into M-ary symbols transmitted by the specified k subcarriers. For example, if the number of subcarriers of each subgroup is equal to 4 ($L = 4$) and the number of active subcarriers is equal to 2 ($k = 2$), the p_1 bits will be equal to:

$$p_1 = \left\lfloor \log_2 \left(C_2^4 \right) \right\rfloor = \left\lfloor \log_2 \left(\frac{4!}{(4-2)!2!} \right) \right\rfloor = \left\lfloor \log_2 (6) \right\rfloor = 2,$$

where $\lfloor z \rfloor$ refers to the largest integer less than or equal to z. The corresponding lookup table of p_1 bits, index $I_{p_1}^{(r)}$ values, and the position of active subcarriers are shown in Table I, where the L subcarriers of the r-th subgroup are expressed as follows:

$$\left[S_1^{(r)}, S_2^{(r)}, \cdots, S_L^{(r)} \right],\ r = 1, \cdots, g,$$

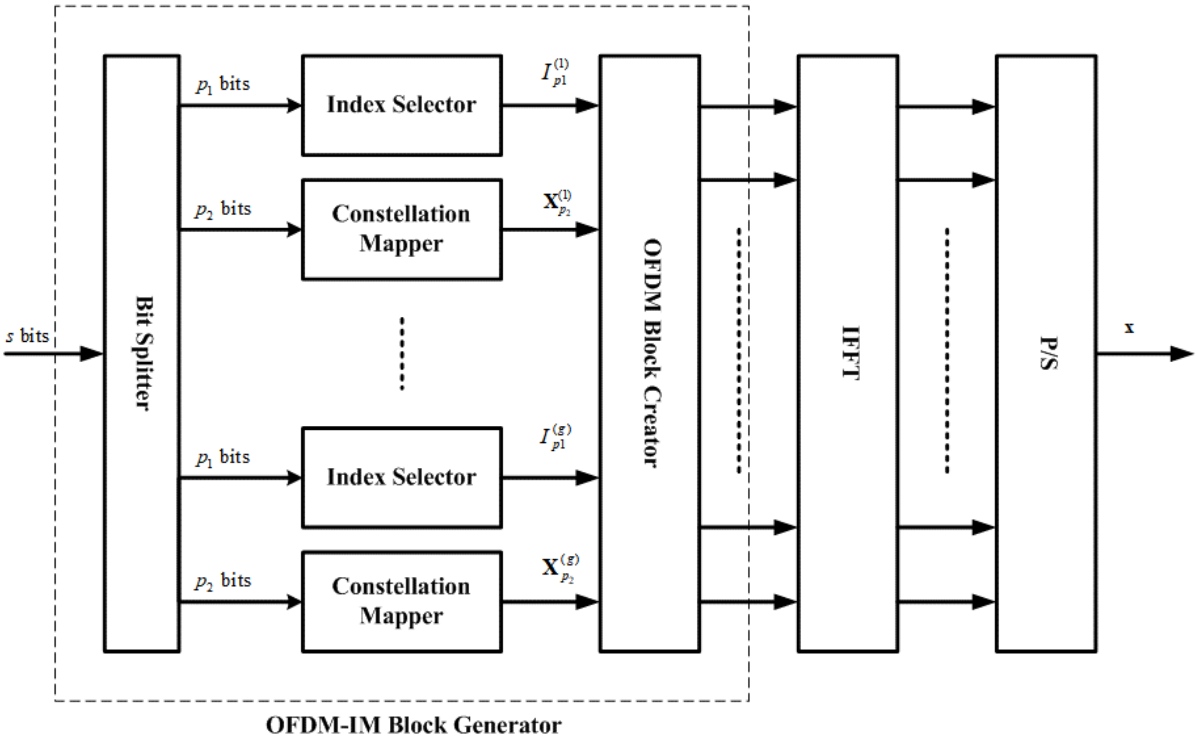

Fig. 1. Block diagram of an OFDM-IM system.

TABLE I: LOOKUP TABLE OF OFDM-IM SYSTEMS

2 bits (p_1)	Index Value ($I_{p_1}^{(r)}$)	The Position of Active Subcarriers $\left[S_1^{(r)}, S_2^{(r)}, S_3^{(r)}, S_4^{(r)}\right]$
[0,0]	[1,2]	$\left[S_1^{(g)}, S_2^{(g)}, 0, 0\right]$
[0,1]	[2,3]	$\left[0, S_2^{(g)}, S_3^{(g)}, 0\right]$
[1,0]	[3,4]	$\left[0, 0, S_3^{(g)}, S_4^{(g)}\right]$
[1,1]	[1,4]	$\left[S_1^{(g)}, 0, 0, S_4^{(g)}\right]$

if the *j*-th subcarrier $S_j^{(r)}$ is not active, then $S_j^{(r)} = 0$. Furthermore, the input information bits of the constellation mapper, $p_2 = k \log_2 M$ bits, are determined using the number of active subcarriers (*k*) and the *M*-ary digital modulation technology, which are then transformed into *k* symbols after *M*-ary digital modulation technology.

The OFDM block creator will sequentially mount the *p* symbol sequence modulated by the constellation mapper to each group based on the index value ($I_{p_1}^{(r)}$) of each group's active subcarriers. After an inverse fast Fourier transform (IFFT), the output results of the OFDM block creator are converted into a time-domain signal as follows:

$$x_m = \frac{1}{\sqrt{N}} \sum_{i=1}^{N-1} X_i e^{j\frac{2\pi i m}{N}}, \ m = 0, 1, \cdots, N-1 \tag{3}$$

Then, the time-domain signal is transformed into a serial OFDM signal using a parallel to serial converter (P/S converter). The PAPR is used to evaluate the performance of the power amplifiers [1]. For any signal, the PAPR is defined as the ratio of the maximum instantaneous power of the signal to the average power. The mathematical formula of the OFDM-IM signal's PAPR can be expressed as follows:

$$\mathrm{PAPR}(x) = \frac{\max\limits_{m=0,1,\cdots,N-1} |x_m|^2}{E\left[|x_m|^2\right]} \tag{4}$$

where $E[v]$ is the expected value of *v*.

Generally, the Complementary Cumulative Distribution Function (CCDF) of the PAPR is often used to evaluate the performance of improving the PAPR, and its definition is explained as follows. Given a reference value of the PAPR ($PAPR_0$), the CCDF of the PAPR refers to the probability that the PAPR of the transmitted signal is greater than $PAPR_0$, and its mathematical formula can be expressed as follows:

$$\mathrm{CCDF}(PAPR_0) = \mathrm{P_r}(PAPR > PAPR_0) \tag{5}$$

When the PAPR of the signal is too high, it not only reduces the performance of the high-power amplifier but also increases the complexity of the analog/digital converter. For the OFDM-IM system, the subcarriers are no longer independent of each other because the active subcarriers are prespecified by the input data bits; this phenomenon also results in new research on reducing the PAPR technological challenge.

B. OFDM-IM Using Selective Mapping Technique

Fig. 2 shows the transmitter of an OFDM-IM system based on the selective mapping technique (OFDM-IM-SLM). In Fig. 2, the SLM technique randomly generates *W* scrambled phase signals of length *N* as follows:

$$b^i = \left[b_0^i, b_1^i, \cdots, b_{N-1}^i\right], \ i = 1, 2, \cdots, W,$$

where the content value of each scrambled phase signal is $b_l^i = e^{j\varphi_l}$, $l = 0, 1, \cdots, N-1$ and $\varphi_l \in (0, 2\pi]$. Following that, the SLM technique multiplies the generated signals of the OFDM-IM block generator by *W* scrambled phase signals of length *N* to generate *W*-multiplied signals, which are then converted into *W* time-domain candidate signals using the IFFT x^l, $l = 1, 2, \cdots, W$. The SLM technology selects the signal with the smallest PAPR from the *W* time-domain candidate signals as the transmission signal of the OFDM-IM system.

However, the SLM technique is a multiple signal representation (MSR) technique, in which the MSR technique generates multiple candidate signals by changing the phase of the input data, and selecting the signal with the smallest PAPR from the candidate signals as the transmission signal of the communication system used. The proposed method of reference [9] indicates the position of the active subcarrier through a preset corresponding lookup table and then mounts the data to the designated an active subcarrier, and then interleave the mounted signal through interleaving. Note that the inter-leaver can gain more diversity. The signal after arrayers generates *W* permutation signals, which are multiplied by the disturbed phase signals and then converted into candidate signals by IFFT, and the signal to be transmitted is obtained from these candidate signals. The signal with the lowest PAPR was selected for transmission.

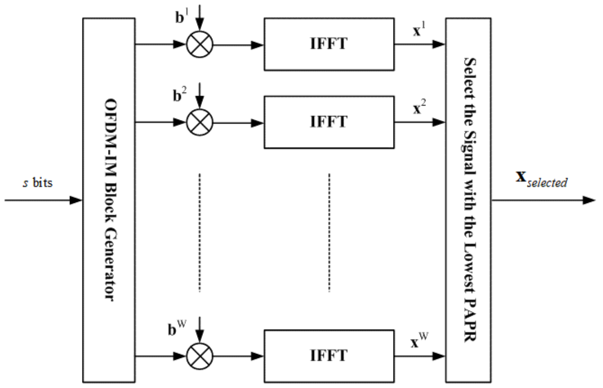

Fig. 2. Block diagram of an OFDM-IM system with SLM

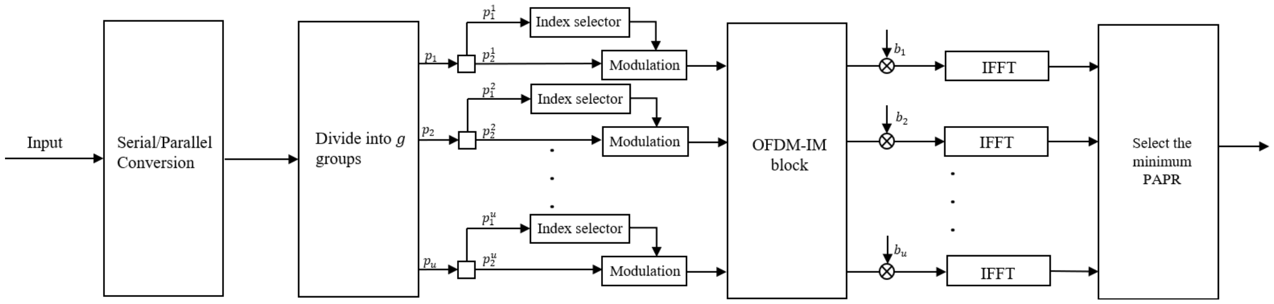

Fig. 3. Block diagram of the proposed method

III. PROPOSED METHOD

Fig. 3 shows the block diagram of the modified OFDM-IM-SLM, which uses the Hamming weight distribution of RM code to build the corresponding lookup table; thus, the input information p_1 bits of the corresponding lookup table have an error correction capability. However, this study also uses Polar codes to perform data protection on the input information p_2 bits.

A RM code is a linear block code that can be used for any order (r) and can be expressed as RM (r,m). Its mathematical expression is as follows

$$\left[n=2^m,\ b=\sum_{i=0}^{r}\binom{m}{r},\ d=2^{m-r} \right] \qquad (6)$$

where r is from 0 to m, b represents the number of data bits [14], and d represents the minimum Hamming distance. The generator matrix of RM codes can be expressed as follows:

$$\mathbf{G}_{\mathrm{RM}(r,m)}=\begin{bmatrix} 1 \\ \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_1\mathbf{x}_2 \\ \mathbf{x}_1\mathbf{x}_3 \\ \vdots \\ \mathbf{x}_{m-1}\mathbf{x}_m \\ \vdots \\ \mathbf{x}_{m-r+1}\cdots\mathbf{x}_m \end{bmatrix} \qquad (7)$$

where the total number of bits is 2^m, and the generator matrix uses the Kronecker product to generate a $k\times n$ matrix, and its mathematical formula can be expressed as follows:

$$\mathbf{G}=\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{\otimes m} \qquad (8)$$

By using a RM code $\mathbf{G}_{\mathrm{RM}(2,3)}$ of order 2 as an example, its generator matrix will be as follows:

$$\mathbf{G}_{\mathrm{RM}(2,3)}=\begin{bmatrix} 1&1&1&1&1&1&1&1 \\ 0&1&0&1&0&1&0&1 \\ 0&0&1&1&0&0&1&1 \\ 0&0&0&0&1&1&1&1 \\ 0&0&0&1&0&0&0&1 \\ 0&0&0&0&0&1&0&1 \\ 0&0&0&0&0&0&1&1 \end{bmatrix}=\begin{bmatrix} \mathbf{1} \\ \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_1\mathbf{x}_2 \\ \mathbf{x}_1\mathbf{x}_3 \\ \mathbf{x}_2\mathbf{x}_3 \end{bmatrix}$$

when m = 3, there will be three variables $\{\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3\}$, and the codeword will be extracted from the generator matrix of the RM code, where the minimum Hamming distance $d=2$, and the mathematical formula for the minimum Hamming distance are as follows:

$$d\ge 2t+1 \text{ and } t=\left\lfloor \frac{d-1}{2} \right\rfloor \qquad (9)$$

For the structure of the generator matrix $\mathbf{G}_{P(b,n)}$ of Polar codes P(b, n) [15], there will be a code length of 2^m and m greater than 1. The generator matrix $\mathbf{G}_{P(b,n)}$ is mainly used to extract the submatrix generated $\mathbf{G}_{\mathrm{RM}(m,m)}$, and how to extract the submatrix is determined based on the vector $\mathbf{y}_{2a}=\left[y_{2a,1},y_{2a,2},\cdots,y_{2a,n}\right]$, where the vector content $\mathbf{y}_{2a}$ is determined using the following recursive structure:

$$y_{2a,b}=\begin{cases} 2y_{a,b}-y_{a,b}^2, & \text{for } 1\le b\le a \\ y_{a,b-a}^2, & \text{for } a+1\le b\le 2a \end{cases} \qquad (10)$$

where $a=1,2,\cdots 2^{m-1}$ and $y_{1,1}=0.5$. The generator matrix $\mathbf{G}_{P(b,n)}$ of the Polar code P(b, n) is obtained by sequentially extracting the b column matrices of $\mathbf{G}_{\mathrm{RM}(m,m)}$ according to the content value of the vector $\mathbf{y}_{2a}$. By using the Polar code P(2,4) as an example, its generator matrix $\mathbf{G}_{P(2,4)}$ can be obtained using the following submatrix:

$$\mathbf{G}_{\mathrm{RM}(2,2)}=\begin{bmatrix} 1&0&0&0 \\ 1&1&0&0 \\ 1&0&1&0 \\ 1&1&1&1 \end{bmatrix},$$

and the vector $\mathbf{y}_{2a}$ using Eq. (10), we can obtain

$$\mathbf{y}_{2a} = [0.9375, 0.4375, 0.5625, 0.0625]$$

According to the content value from small to large, the position coordinates can be obtained as [4 2 3 1]. The generator matrix $\mathbf{G}_{P(2,4)}$ can be constructed by extracting two columns (b = 2) from the generator matrix $\mathbf{G}_{\mathrm{RM}(2,2)}$, where the extracted columns are the second column and the fourth column of the generator matrix $\mathbf{G}_{\mathrm{RM}(2,2)}$ (according to $y_{1,1} = 4$ and $y_{1,2} = 2$). After that, the generator matrix $\mathbf{G}_{P(2,4)}$ can be written as follows:

$$\mathbf{G}_{P(2,4)} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

The parameters of Polar codes vary according to the number of active subcarriers (k) and the number of groups (g). The number of information bits of the Polar codes, P, is equal to the number of active subcarriers (k) multiplied by the number of groups (g). For example, the value of P is equal to 128 when $k = 4$ and $g = 16$. Note that the value of P is equal to $\log_2(M) \times (k \times g)$ when the digital modulation of the OFDM-IM system is M-ary PSK or M-ary QAM.

IV. SIMULATION RESULTS

In this study, the traditional OFDM-IM technique, the OFDM-IM-SLM with the control group [9], and the method proposed in this paper (modified OFDM-IM-SLM) are evaluated for PAPR reduction performance. The number of candidate signals (W) is 8 and the value of the phase sets is {±1, ±j}. Assuming that the digital modulation technique is QPSK modulation, the number of active subcarriers (k) is 4 and the number of groups (g) is 16.

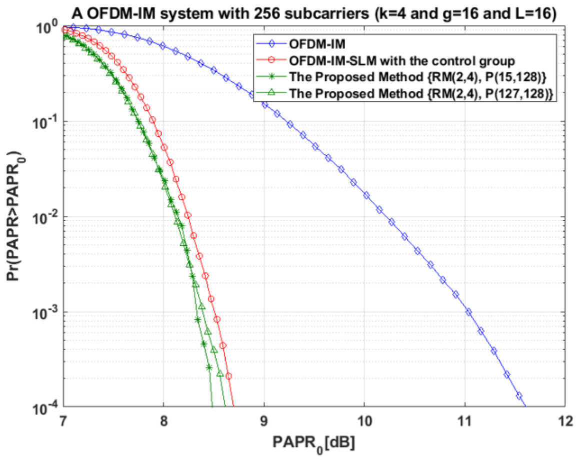

Fig. 4. The PAPR performance comparison between the OFDM-IM-SLM with the control group and the modified OFDM-IM-SLM when N = 256

Fig. 4 shows a comparison of the PAPR performance of the traditional OFDM-IM system, the OFDM-IM-SLM with the control group [9], and the modified OFDM-IM-SLM system proposed in this study. The number of subcarriers (N) is 256 (Fig. 4). The linear blocks of p_1 and p_2 use the second order RM codes with m = 4 and the Polar code with P(15,128) and P(127,128), respectively. As $\Pr(PAPR > PAPR_0) = 10^{-4}$, the OFDM-IM-SLM with the control group, the traditional OFDM-IM-SLM, and the traditional OFDM-IM are 8.63 dB and 11.6 dB, respectively (Fig. 4). Moreover, when the Polar codes are b = 15 and b = 127, the PAPR reduction method proposed in this study reduces by 0.17 dB and 0.03 dB, respectively.

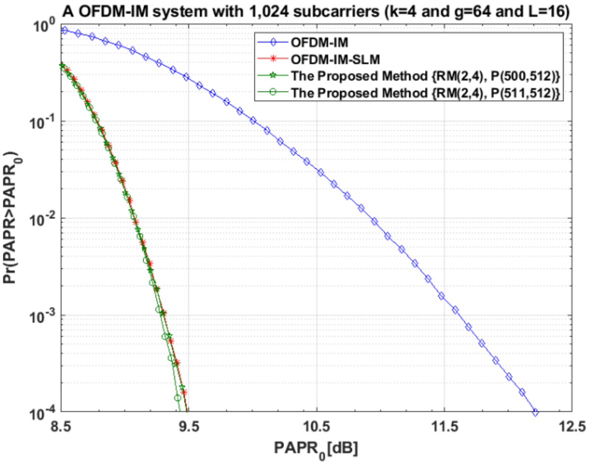

Fig. 5. The PAPR performance comparison between the traditional OFDM-IM-SLM and the modified OFDM-IM-SLM when N = 1,024

Fig. 5 compares the PAPR performance of the traditional OFDM-IM system, the traditional OFDM-IM-SLM system, and the modified OFDM-IM-SLM system proposed in this study. The number of subcarriers is 1,024 (Fig. 5). The linear blocks of p_1 and p_2 use the second order RM codes with m = 4 and the Polar code with P(500, 512) and P(511, 512), respectively. As $\Pr(PAPR > PAPR_0) = 10^{-4}$, the traditional OFDM-IM-SLM and the traditional OFDM-IM are 9.46 dB and 12.1 dB, respectively (Fig. 5). Moreover, the PAPR reduction method proposed in this study reduces by 0.01 dB and 0.05 dB when the Polar codes are b = 500 and b = 511, respectively. The proposed method uses two Polar codes with different error correction capabilities (Fig. 4), and both can achieve the same PAPR improvement performance as OFDM-IM. Similarly, when the number of subcarriers increases, the proposed method uses Polar codes with two error correction capabilities (Fig. 5), which can also achieve the same PAPR improvement performance as OFDM-IM.

V. CONCLUSION

In this study, we propose an improved SLM technique used in the OFDM-IM system. The proposed method uses the Hamming weight distribution of the specific order RM codes to construct the lookup table of the OFDM-IM system. Thus, the information for selecting active subcarriers has error correction capability because of the RM code. The proposed method also uses Polar codes as channel codes to enable the transmitted information to have error correction capability. The simulation results show that the proposed method has better PAPR performance than the original OFDM-IM and OFDM-IM-SLM systems, and the proposed method can detect and correct errors for the data to be transmitted.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

H. Y. Liang conducted the research; H. W. Liu analyzed the data; all authors wrote the paper; all authors had approved the final version.

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